Measuring Consumer Freedom*

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Abstract

This paper axiomatizes an index of consumer freedom of choice and applies it to U.S. consumer data from 2004 to 2017. The Rényi (1961) entropy arises naturally as an index of the residual of welfare that remains after accounting for total consumption, the price level, and inequality. We show that this index can be interpreted as the freedom of choice of a representative consumer allocating a dollar of expenditure to goods. When applied to data, we find that the freedom of choice index was stagnant between 2004 and 2009 but substantially increased thereafter. This increase is driven by an increase in the total number of goods consumed as well as a more even distribution of expenditure across goods.
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1 Introduction

The gross domestic product is often used by the public as a crude measure of the welfare of a country. However, it is well known that total income does not account for important aspects of the welfare of a society. Thus, other indices are needed to complete the picture. Inequality indices account for differences in income (Atkinson, 1970; Ben-Porath & Gilboa, 1994; Chakravarty, 1988; Dalton, 1920; Ebert, 1988; Gini, 1921; Kolm, 1976), consumption quantity indices for the quantities consumed (Sato, 1976; Vartia, 1976), and price inequality estimates account for potential heterogeneity of purchasing power (Broda & Romalis, 2009; Handbury & Weinstein, 2015; Jaravel, 2018; Pisano & Stella, 2015). We axiomatically examine the question of what information a welfare index based on consumption data can track if total consumption, the price level, and household inequality are irrelevant (or accounted for via other indices). Aside from imposing that the index is invariant in the aforementioned aspects, we only impose continuity, an independence condition, and symmetry across goods to characterize the index. We characterize an entropy index of the diversity of the expenditure allocations of consumers. A higher entropy means that consumers' choices are quantitatively more diverse in the sense of the number of products purchased or how even the expenditure was distributed across these products. We then show that this index can be interpreted as an index of consumer freedom of choice. For this, we use a different setup to characterize a generalization of the Suppes (1996) measure of freedom of choice and show that it yields an identical index as our first characterization. Thus, our index can be interpreted as the freedom of choice of a representative consumer to allocate a dollar of expenditure.

This index can be readily applied to consumption expenditure data. We therefore provide in the empirical part of the paper the first estimate of consumer freedom of choice for U.S. households from detailed household level consumption data. To our knowledge, this is also the first empirical application of an index suggested by the freedom of choice literature.[1] We measure freedom of choice for the U.S. economy between 2004 and 2017 using the Nielsen Consumer Panel. We obtain that over the duration from 2004 to 2009, the freedom of choice index was largely constant. However, from 2010 to 2017 we observe in our dataset a striking increase in the freedom of choice index. For comparison, the increase in entropy that

[1] For a survey of the freedom of choice literature, see (Dowding & van Hees, 2009).

we observe is equal to that of an increase from uniformly distributed expenditure on 80 000 products to uniformly distributed expenditure on 100 000 products. We analyze this increase more closely and conclude that the increase is due to an increase in the number of options and a more even distribution of expenditure across goods by different households. We exclude alternative explanations such as changes in the distribution of prices, changes in the demographic composition of our sample, and the love of variety of the utility functions of individual households.

We show that when estimating the consumer freedom of choice index for subgroups of our sample of households, the freedom of choice index differs across age subgroups and racial subgroups but not as strongly across income groups. Thus, the quantitative diversity of expenditure allocation is similar for rich and poor households. Markets provide both rich and poor households with a similarly high diversity of choice. However, we find that the main limitation to freedom from income is the predetermination of consumers' choices by their income class. Thus, under the assumption that consumers are not morally responsible for the income they earn, freedom in a more general sense is limited by income determining the choices of households. Already under relatively coarse income categories the degree to which income is predictive of consumption choices of two-person households yields a decrease in freedom equivalent to about one third of the increase of our freedom of choice index that we observed in our 14 year sample period.

The paper proceeds as follows. We first introduce the freedom of choice index in Section 2. In Section 3 we axiomatize the index. We estimate the freedom of choice index on data described in Section 4. The results are presented in Section 5. In particular, we show evidence of a substantial increase in the freedom of choice index in Subsection 5.1, we discuss demographic determinants of freedom of choice in Subsection 5.2 with a special focus on the role of income. Section 6 discusses our results.

2 Consumption Freedom of Choice Index Measure

Theil (1965, 1967) first connected demand analysis with information-theoretic concepts. When being provided with a dataset, information theoretic measures may provide a variety of indices of different aspects of the economy. The entropy of the income distribution can be used to measure income (in-)equality, logarithmic price indices can be given an information-theoretic interpretation (Theil, 1965) and the Herfindahl-Hirschman index of market concentration is a monotone transformation of the Rényi (1961) entropy applied to market shares. Suppes (1996) suggests measuring the freedom of choice of consumers using the entropy rate of the choice frequencies.

The freedom of choice index literature (Dowding & van Hees, 2009) has developed measures to capture freedom of choice an individual has when facing an opportunity set. In this literature, Suppes (1996) suggests to use the Shannon (1948) entropy rate of the choices made by an individual as a measure of freedom of choice. The Shannon entropy is defined as follows:

$$H(p) = - \sum_{x \in supp(p)} p(x) \ln(p(x)) \tag{1}$$

where p is a probability mass function over a finite set $\mathcal{S}$ and $supp(p)$ is the support of p, i.e., $x \in supp(p) \Leftrightarrow p(x) > 0$. The entropy measures a particular tradeoff between the size of the support and how evenly the elements of the support are distributed. In case the support is completely evenly distributed, then the entropy is equal to the logarithm of the cardinality of the support. In case one alternative has almost probability mass one, then the entropy is close to zero. More generally, whenever an element of the support has a higher probability than a second element, then redistributing some of the probability mass from the former to the latter increases the entropy.

The Shannon entropy rate of choices is now defined as the entropy measure applied to the long-run probability with which each element from $\mathcal{S}$ is drawn if choices are made repeatedly by an individual from $\mathcal{S}$. The intuition behind the Suppes (1996) measure is simple; the freedom of choice is larger, the more alternative options are chosen and the more evenly distributed the choice probabilities are.

Naturally, this measure does not capture the diverse philosophical ideas (e.g., qualitative diversity, Nehring and Puppe (2009); control Ahlert (2010), Rommeswinkel (2019a), Sher (2018); available opportunities Pattanaik and Xu (1990); reasonableness of opportunities Jones and Sugden (1982); rights, Pattanaik (1994); etc.) associated with freedom of choice, many of which have been accounted for in other measures of freedom of choice. However, the theoretical freedom of choice literature has remained without much empirical impact at least partly due to the resulting complexity that comes with accounting for all of these aspects. In our axiomatic analysis, we show that the Suppes (1996) measure is well suited for empirical applications.

Perhaps the biggest issue with the Suppes (1996) measure is that it treats all choices symmetrically. One could argue that for example the choice between two different cartons of egg provides less freedom of choice than the choice between a carton of eggs and a carton of milk as the latter choice involves qualitatively more diverse options. However, qualitative diversity may also reduce freedom of choice; consider the choice between a carton of free-range eggs versus a carton of battery eggs. If we replace the battery eggs by rotten battery eggs, the opportunities are clearly more diverse but freedom of choice has arguably decreased. Further, measures of qualitative diversity (e.g., Nehring & Puppe, 2002) involve a large number of parameters and it is not clear how these could be extracted from consumption data. We below show that applying the Suppes (1996) measure to choices over dollar allocations instead of choices between goods is a sensible compromise that maintains empirical tractability while making the symmetry property of the entropy less unappealing.

3 Axiomatization

There are potentially many dimensions by which consumption data can be evaluated to measure welfare. We examine a welfare index in which we ignore all aspects that are often associated with welfare. In particular, we axiomatize the index such that total quantities, inflation, and inequality are irrelevant. Our index is therefore an answer to the question of what a policy maker may care about after total consumption, inequality, and the price level have been accounted for in a satisfactory manner. We show that any index that is subgroup decomposable and symmetric across goods measures the quantitative diversity of the expenditure across goods as measured by a Rényi (1961) entropy. We then show that this index can be interpreted as the freedom of choice of spending a dollar on one good out of a set of goods that is exercised by a representative agent.[2]

The Nielsen Consumer panel specifies for every shopping trip of a household what items have been purchased at what price. Before starting the axiomatic derivation of our index, we reduce this data to information about how much a consumer spent on which product within a year and

[2]It is important to note that "representative agent" in the context of freedom of choice does not imply the maximization of a single preference relation, see for example Nehring and Puppe (1999).

what quantity the consumer obtained of this product. Thus, we assume that an analyst receives shopping data that specifies for every good g and every household h a quantity purchased q_g^h and the expenditure on the good, e_g^h for an entire year. For simplicity, we denote the vector of household h's consumption (expenditure) by q^h (e^h) and by q_g (e_g) the vector of quantities consumed (expenditure spent) by households on good g. For a finite set of goods $\mathcal{G}$ and a finite set of households $\mathcal{H}$, let $\mathcal{D} \subset (\mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0})^{\#\mathcal{G} \cdot \#\mathcal{H}}$ be the set of possible data sets (q, e) where we assume that quantities of a good are strictly positive if and only if expenditure is strictly positive. We endow this set with the (subspace topology of the) product topology of the real numbers. Any index that assigns every possible dataset a real number naturally generates a preorder on datasets and in reverse a binary relation $\succsim$ on $\mathcal{D}$ is said to be represented by an index U whenever $U(q, e) \geq U(q', e')$ if and only if $(q, e) \succsim (q', e')$.

Classically, entropy measures are often characterized by assuming that a real valued representation exists and imposing an additivity property on this function. Instead, we will follow the order-based approach together with separability properties, which is more standard in economics.[3] For order-based axiomatizations, the first two axioms are standard.

Definition 1 (Weak Order)**.** A binary relation $\succsim$ on a set fulfills weak order if it is complete and transitive.

Definition 2 (Continuity)**.** A binary relation $\succsim$ on a set fulfills continuity if for all elements the weakly upper and weakly lower sets are closed in the relevant topology.

Since there already exist a large number of quantity, price, and inequality indices, with our next axioms we assume that our index is homogeneous of degree zero in quantities and the price level, and invariant in inequality.

Definition 3 (Quantity Invariance)**.** A binary relation $\succsim$ on $\mathcal{D}$ fulfills quantity invariance if for all positive real numbers α and all $(q, e) \in \mathcal{D}$, we have that $(q, e) \sim (\alpha q, \alpha e)$.

With this axiom, we assume that increasing total consumption linearly does not change the index.

[3] Mathematically closest to this paper's characterization is Rommeswinkel (2019b), which characterizes the entropy as a procedural value. The richer structure of primitives in Rommeswinkel (2019b) allows for a simpler proof structure.

Definition 4 (Inflation Invariance)**.** A binary relation $\succsim$ on $\mathcal{D}$ fulfills inflation invariance if for all positive real numbers α and all $(q, e) \in \mathcal{D}$, we have that $(q, e) \sim (q, \alpha e)$.

With this axiom, we assume that multiplying all prices by a common factor does not influence the index. Thus, the index is invariant to the first-round effects of inflation, though it of course may be indirectly influenced by inflation via changes demand. This also means that we ignore that higher prices mean that consumers spend a larger proportion of their income on the goods. Again, this would be captured by other indices such as the average household savings.

Let $(0_G q, 0_G e)$ denote the data in which all goods in $G \subseteq \mathcal{G}$ have zero expenditure and quantity but all other quantities and expenditure are the same as in (q, e).

Definition 5 (Weak Subgroup Decomposability)**.** A binary relation $\succsim$ on $\mathcal{D}$ fulfills weak subgroup decomposability if for all (q, e) and (q', e') that are identical on a subset of goods G and with an equal total expenditure, we have that,

$$(q, e) \succsim (q', e') \quad \Leftrightarrow \quad (0_G q, 0_G e) \succsim (0_G q', 0_G e') \tag{2}$$

Weak Subgroup Decomposability is an assumption that we would (at least implicitly) have to make anyways in case that we only observe a subset of all goods consumed. Whenever we estimate an index without having full access to the entire consumption of households, we nonetheless presume that the calculation of the index on the partial data is meaningful. Weak Subgroup Decomposability makes this precise by stating that *ceteris paribus*, differences in quantity and expenditure on a subset of goods can be evaluated by calculating the index for that subset. The ceteris paribus clause makes two impositions: expenditure and quantity of all other goods need to be identical and the total expenditure across all goods needs to be identical as well. The latter assumption guarantees that the relative importance of the subset of goods is the same.

Lemma 1 (Share Representation)**.** *If a binary relation $\succsim$ fulfills Quantity Invariance, Inflation Invariance, and Weak Subgroup Decomposability, then* $(q, e) \sim \left(\left(\frac{1}{\sum_{h \in \mathcal{H}} q_g^h} q_g \right)_{g \in \mathcal{G}}, \frac{1}{\sum_{h \in \mathcal{H}, g \in \mathcal{G}} e_g^h} e \right)$.

Thus, under the two invariance axioms and Weak Subgroup Decomposability, the ranking position of (q, e) in the index does not depend on the total quantity of each good consumed or the quantity consumed of one good compared to other goods. It may however depend on the relative share each consumer obtains of each good (for example the inequality of the quantity of a good across consumers). With respect to expenditure, an index fulfilling the two invariance conditions may depend on both the distribution of expenditure across goods and consumers. We therefore next impose that the inequality of both quantities and expenditure of the consumers does not matter.

We say that (q, e) is obtained from (q', e') by redistributing some of the expenditure and quantity of good g from individual j to individual i whenever the following conditions hold: First, (q, e) and (q', e') are identical on all but individual i's and j's consumption and expenditure of g. Second $q_{i,g} > q'_{i,g}$, i.e., the quantity consumed of good g increases for i. Third, $q_{i,g} + q_{j,g} = q'_{i,g} + q'_{j,g}$ and $e_{i,g} + e_{j,g} = e'_{i,g} + e'_{j,g}$, i.e., total quantities and expenditure remain the same. Fourth, $e'_{j,g} = e_{j,g}\frac{q'_{j,g}}{q_{j,g}}$, i.e., individual j still pays the same price for the remaining consumption.

Definition 6 (Inequality Invariance)**.** A relation $\succsim$ on $\mathcal{D}$ fulfills inequality invariance if whenever (q, e) is obtained from (q', e') by redistributing some of the quantity and expenditure of good g from j to i, then $(q, e) \sim (q', e')$.

The axiom states that redistributing some of j's expenditure and quantity of a good to individual i has no effect on the index. This implies invariance of our index both to inequality in quantities consumed and to inequality in the prices paid for the products as measured by Pisano and Stella (2015). Naturally, there could also be inequality in freedom of choice and limitations of freedom of choice due to inequality, issues that we return to in Sections 5.2 and 5.3.

Let $s_{(q,e)}$ be the mass function of expenditure e, i.e.,

$$s_{(q,e)}(g) = \frac{\sum_{c \in \mathcal{C}} e_{cg}}{\sum_{c \in \mathcal{C}, g \in \mathcal{G}} e_{cg}}. \tag{3}$$

Let ΔS denote the set of all mass functions on a finite set S.

Lemma 2 (Expenditure Share Representation)**.** *If a binary relation $\succsim$ fulfills Quantity Invariance, Inflation Invariance, Inequality Invariance and Weak Sub-*

group Decomposability, then there exists a binary relation $\succsim^*$ on $\Delta\mathcal{G}$ such that $(q, e) \succsim (q', e')$ if and only if $s_{q,e} \succsim^* s_{q',e'}$.

Without further conditions, the analyst may judge a higher expenditure share on a particular good as contributing more to the index than another good. To avoid this, we impose the following condition.

Definition 7 (Goods Symmetry)**.** A relation $\succsim$ on a set fulfills goods symmetry if whenever any two elements of the set are permutations of another with respect to goods, then they are indifferent.

This last condition states that the index remains neutral across goods. A higher expenditure or quantity of a particular good is not preferable to a higher expenditure or quantity of another good. This is plausible if the analyst is agnostic about how different goods affect the well-being of a consumer.

Rényi (1961) introduced the following generalization of the Shannon (1948) entropy.

Definition 8 (Rényi (1961) Entropy)**.** The Rényi (1961) entropy of degree $r > 0$ of a probability mass function $s \in \Delta\mathcal{G}$ is

$$H_r(s) = \begin{cases} -\sum_{g\in\mathcal{G}} s(g) \ln s(g) & r = 1 \\ -\frac{1}{1-r} \ln \sum_{g\in\mathcal{G}} s(g)^r & \text{else.} \end{cases} \tag{4}$$

This entropy measure shares similar properties as the Shannon (1948) entropy; it increases in the size of the support and how close the mass function is to the uniform distribution. However, the parameter r controls how much importance the entropy attaches to the size of the support versus the uniformity of the mass function. As $r \to 0$, $H_r(s)$ converges to a counting measure of the number of elements in the support of s while as $r \to \infty$, $H_r(s)$ converges to a decreasing function of the maximum probability of all the elements in the support.

Since none of our axioms restrict the $\succsim$ to be increasing or decreasing in support and uniformity of expenditure, the characterized index can be either increasing or decreasing in the entropy:

Theorem 1 (Entropy Representation)**.** *Let $\succsim$ be a binary relation on a set of possible data sets $\mathcal{D}$ with at least three pairs of essential goods. Then the following statements are equivalent:*

1. $\succsim$ *fulfills Weak Order, Quantity Invariance, Inflation Invariance, Inequality Irrelevance, Weak Subgroup Decomposability, and Symmetry.*

2. *There exists $r > 0$ such that $\succsim$ can be represented by a Rényi (1961) entropy $\pm H_r$ applied to the expenditure shares.*

This characterization is in some sense a negative one; we have characterized the index by stating that it is invariant in total quantities, total expenditure, and inequality across households. The remaining conditions can be seen as technical conditions that come with the creation of indices from consumption data.

However, it would be desirable to also characterize the index starting from a concept of what the analyst wants to measure and impose axioms that allow for this. We therefore provide a second, alternative interpretation of the index. Suppose a representative consumer decides on allocating a dollar of expenditure to one good of a finite set of goods $\mathcal{G}$. Let the analyst be in possession of data on the probability with which the consumer allocates the dollar to each good. The analyst wants to use this data in order to determine the degree of freedom of choice the consumer exercises. The analyst therefore ranks elements of $\Delta\mathcal{G}$, the set of probability mass functions over goods. Thus, for a mass function $p \in \Delta\mathcal{G}$, we denote by $p(g)$ the probability that the consumer will choose to spend the dollar on good $g \in \mathcal{G}$. The mass function that yields good g with certainty is denoted by $[g]$.

The preference relation of the policy maker on $\Delta\mathcal{G}$ is denoted by $\succsim$. A pair of goods $g, g' \in \mathcal{G}$ is called essential if there exist μ, λ such that $\mu[g] \oplus (1-\mu)[g'] \succ \lambda[g] \oplus (1-\lambda)[g']$. Let $\oplus$ be the standard mixture operator. If $p, q \in \Delta\mathcal{G}$, then for any $\mu \in [0,1]$, $\mu p \oplus (1-\mu)q = r$ is the distribution such that $r(g) = \mu p(g) + (1-\mu)q(g)$ for all $g \in \mathcal{G}$.

The independence axiom of von Neumann and Morgenstern (1944) states that preferences are invariant with respect to identical mixtures. We restrict the independence axiom to the case of disjoint supports for reasons we elaborate on in a moment. For this, we define the support of a distribution p, $supp(p) = \{g \in \mathcal{G} : p(g) > 0\}$ as the set of outcomes reached with positive probability. p and q have disjoint supports whenever these sets are disjoint.

Definition 9 (Disjoint Independence)**.** A binary relation $\succsim$ fulfills disjoint independence if for all p, q disjoint from r, we have:

$$p \succsim q \quad \Leftrightarrow \quad \alpha p \oplus (1-\alpha) r \succsim \alpha q \oplus (1-\alpha) r \tag{5}$$

Disjoint Independence imposes the von Neumann-Morgenstern independence axiom only on mixtures of distributions with disjoint support. This axiom requires some motivation. Traditionally, the von Neumann-Morgenstern axiomatization of expected utility assumes the independence condition (5) to hold for all lotteries. We only assume that this condition holds for disjoint supports for the following reason.

Suppose in mass function $[apple]$ the representative consumer chooses with certainty to spend the dollar on an apple while in $[banana]$ the consumer chooses with certainty to spend the dollar on a banana. Suppose Symmetry holds such that the analyst ranks both distributions equally. Then the independence axiom – if imposed also on distributions that are not disjoint – would require the analyst to also rank $[apple]$ equal to any mixture between $[apple]$ and $[banana]$. For example, the analyst would be required to rank the situation in which she is certain that the representative consumer spends the dollar on the apple equal to the situation in which the consumer spends the dollar with equal probability on either good. However, the analyst may think that in the situations $[apple]$ and $[banana]$ the representative consumer exercises no meaningful choice but that in the situation $1/2[apple] \oplus 1/2[banana]$ the consumer exercises a meaningful choice. Therefore, independence must be weakened to disjoint independence to allow (though not require) the analyst to rank the mixture of $[apple]$ and $[banana]$ as strictly preferable to either of the two.

Theorem 2 (Mixture Representation)**.** *Let $\succsim$ be a binary relation on the set of probability mass functions $\Delta\mathcal{G}$ with at least four pairs of essential goods. Then the following statements are equivalent:*

1. *$\succsim$ fulfills Weak Order, Continuity, Symmetry, and Disjoint Independence.*
2. *There exists $r > 0$ such that $\succsim$ can be represented by a Rényi (1961) entropy $\pm H_r$ of the probability mass function.*

We have therefore obtained the same generalization of the Suppes (1996) measure of freedom of choice applied to the decision to allocate a dollar of spending to a set of goods.[4] We provide in Appendix A.5 a cardinal consistency axiom that together with the remaining axioms characterizes the Suppes measure uniquely, i.e., guarantees that $r = 1$. However, we do not find this cardinal consistency axiom normatively compelling within the scope of the index on $\mathcal{D}$ or the freedom of choice measure on $\Delta\mathcal{G}$. Generally, a lower value of r states that the analyst cares more about the freedom of choice on goods that receive a low proportion of expenditure while a large value of r emphasizes the freedom of choice over goods that receive a large proportion of expenditure. While the case of $r = 1$ can be seen as a "neutral" intermediate case, there is a priori no reason why a freedom of choice index should not give greater emphasis to let's say freedom of choice over large expenditure goods.

The main motivation for using the Shannon (1948) entropy indeed arises when doing cardinal comparisons with other information theoretic measures such as the mutual information, a point which we will return to in Section 5.3. It is for this reason that in the empirical part of the paper we employ $r = 1$, since reporting our results for all r is infeasible.[5]

It is noteworthy that this is the Suppes measure applied to the decision on how to allocate a dollar of expenditure, not the decision which good to consume. Suppes initially intended the measure to be applied to the choice between arbitrary goods. However, this is normatively not as convincing. First, from a practical perspective, this measure would not be invariant under repackaging of goods, i.e., a change in packaging sizes from one large package of a good to a smaller unit size may affect the measure without any change in expenditure or total consumption. Second, assuming symmetry across goods is much less normatively convincing for the decision whether to purchase an expensive or a cheap good but remains normatively convincing for the decision whether to allocate a dollar to the purchase of a cheap or expensive good. We will however use the entropy measure of freedom of choice applied to quantities in Section 5.1 as a useful robustness check for our results.

Together, the two characterizations yield a powerful result: the index

[4]The proof of Theorem 1 indeed follows from the proof of Theorem 2 after showing that datasets can be represented as expenditure shares. Rommeswinkel (2019b) characterizes the same representation using the classical von Neumann-Morgenstern axioms but weaker assumptions on the mixture sets by using a different method based on certainty equivalents.

[5]At times we will also make use of the $r \to 0$ entropy and results turn out to be qualitatively similar.

that captures the remaining component of welfare after ignoring total quantity of consumption, the price level, and inequality is a representation of the freedom of choice of a representative consumer to choose how to allocate a dollar of expenditure.

4 Data

To estimate our freedom of choice index, we use the Nielsen Consumer Panel, which includes household level consumption data for the years 2004 to 2017. Households track their expenditure via a barcode scanner by scanning the purchased products at home. Naturally, this dataset does not contain certain types of expenditure, for example services or housing. However, according to Nielsen, about 30 percent of the households' total expenditure is accounted for in the dataset. Einav et al. (2010) however note some discrepancies in the reported total amount spent and the amount calculated from the prices of the products scanned. The subgroup decomposability assumption ensures the validity of our index in case some goods are systematically not scanned by consumers. However, biases in our results presented below may occur in case underreporting of purchases of some goods are correlated with time and/or demographics. Products are distinguished via unique product codes (UPCs) of which 1.4 million are found in the dataset. In some cases, a single product has been assigned more than one UPC. For each UPC we have information about the brand name, a description, unit, size, and whether it is a bundle of multiple goods. UPCs are categorized into 11 departments at the top level, about 100 groups at the second highest level, and about 1000 modules at the most detailed level. The Nielsen Consumer Panel is moreover divided into so-called magnet data and non-magnet data. We focus on non-magnet data in order to have a large sample of households.

In addition to the expenditure data, the dataset contains demographic variables of the households such as income, race, age, household size, and education. When comparing the freedom of choice index of different subsamples (years or demographic subgroups) we will argue below that the sample size of the *smallest* subsample is the main limitation. We therefore do not use the demographic variables as provided by the data set but instead a coarser classification.

With respect to age, we classify households into young and old households as follows. A household is young if it belongs to the 50% youngest households by the average of the male and female household head age variable after projecting to make the sample representative. Thus, the YOUNG households make up exactly 50% of the projection weights in every year. The threshold household age varies over time from a minimal age threshold of 50.5 in 2007 to a maximal threshold age of 53 in 2013-2017.

We classify households into two income groups, RICH and POOR households, via their household-size adjusted income. Households report income categorically and therefore we have an upper and a lower bound for the income of every household. For each household, we adjust the boundaries for the number of adults and children by the same method as the U.S. Census Bureau (Fox, 2020).[6] We then calculate the midpoint of the boundaries and classify households into POOR if they are among the 50% households with the lowest adjusted midpoint.

With respect to race, the households are divided into four race subgroups, WHITE, AFRICAN AMERICAN, ASIAN, and OTHER. With respect to income and age, the sample is balanced by construction but with respect to race the sample is heavily unbalanced as evident from Table 1.

Race	Households
White	32552
African American	3892
Asian	877
Other	2256

Table 1: Number of Households for Each Race in 2004

Stability of Demographic Variables

As the freedom of choice index may differ for different subgroups of our sample, it is important for the validity of comparisons of the index over time that the household composition with respect to demographic variables does not change. For example, our results presented in Section 5.2 show that the freedom of choice index is larger for the subsample of YOUNG households than for the subsample of OLD households. If the households of the Nielsen Consumer Panel Data would become younger over time, the freedom of choice index may increase without a real economic effect

[6] Up to two adults with no children, we divide income by the square root of the number of adults. A single parent's income of n children is divided by $(1.8 + (n-1)/2)^{.7}$. All other households' income of m adults and n children are divided by $(m + n/2)^{.7}$.

changing the freedom of choice index. It is therefore important that demographic variables of our sample are stable and that any changes in the demographic composition of the sample reflect actual changes in the composition of households in the U.S..

Figure 1 shows that the average age in the two age groups has remained approximately stable from 2004 until 2017. According to Figure 2, the race composition is approximately stable from 2004 to 2017. As there are still some changes in demographic variables over time, it will nonetheless be important to verify that changes in the freedom of choice index are not driven by these changes in the demographic composition of the households in our sample.

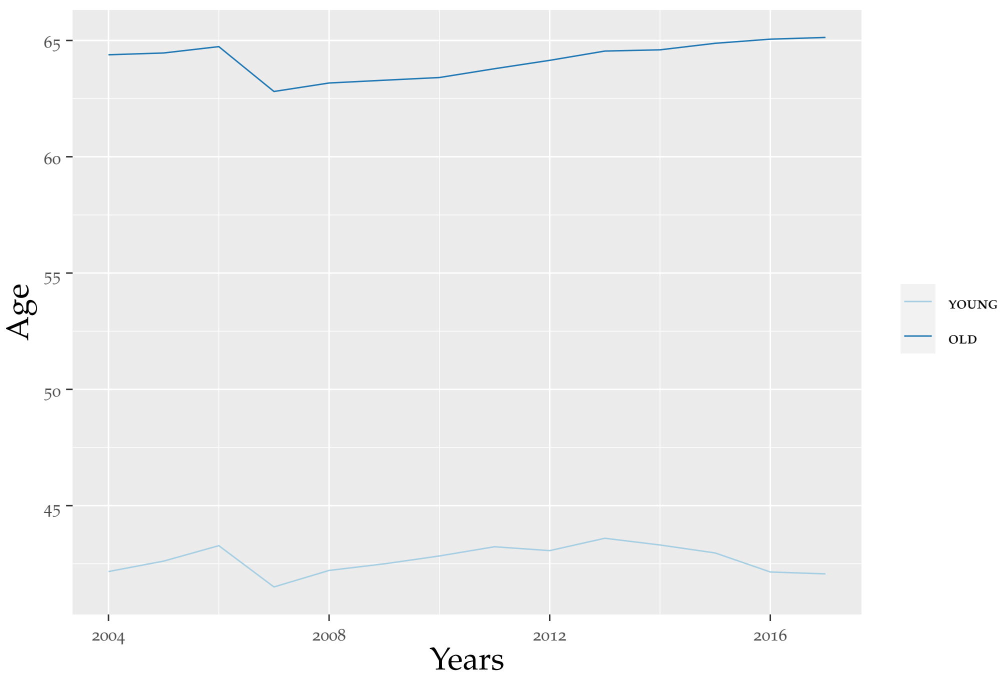

Figure 1: Change in Age Structure Over Time

5 Results

Before analyzing how the freedom of choice index changed over the duration of our data, we make some preliminary remarks and caveats about the estimation. Firstly, the estimate of the entropy is generally

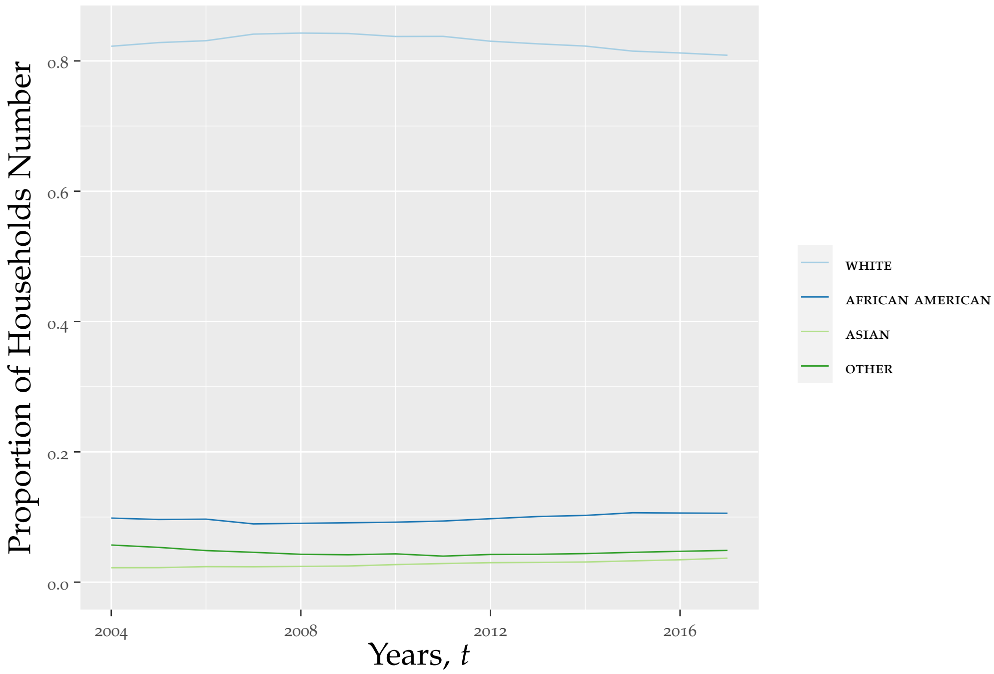

Figure 2: Change in Racial Composition Over Time

increasing in the number of households in the sample. The reason is that if we drop a random houshold from the sample, then the idiosyncratic components of their purchases are no longer observed. Since the entropy measure is increasing in these idiosyncratic components, on average, the result of dropping a household from the dataset is a decrease in entropy.

Figure 3 is obtained by randomly selecting subsamples of various amounts of households from the dataset and estimating the freedom of choice index. The smaller the sample, the lower the estimated index because of the aforementioned effect. Moreover, the fewer the number of households, the noisier the estimates become. An important conclusion to draw from Figure 3 is that if we compare different years or subgroups that have a different number of households, we must adjust for the number of households or account for the functional relationship between sample size and estimates. Since the functional relationship of the freedom of choice index to the number of households is nontrivial, whenever we make comparisons, we instead obtain equally sized samples by dropping households randomly from the year or subgroup of households with a larger number of households, a process we call downsampling. In

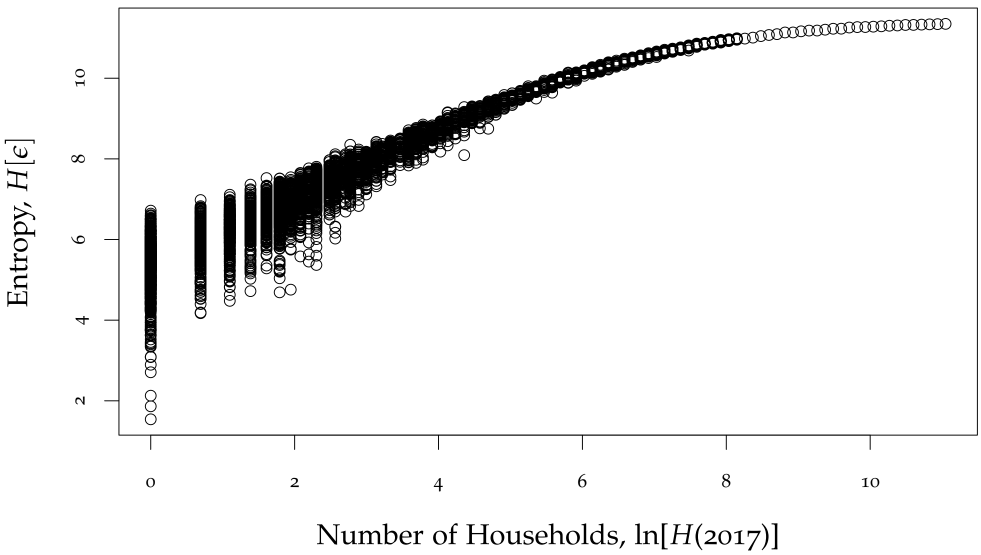

Figure 3: Freedom of Choice Index Depending on Number of Households in 2017

the following, all our results on comparisons across years or groups are obtained via downsampling with n denoting the sample size used for each year or subgroup.

Another observation we can make from Figure 3 is that at sufficiently high sample sizes the estimates have a sufficiently low standard error. Dropping from more than 50 000 households to 10 000 households by random selection has a small, noticeable effect on the index estimate. However, *which* households we drop has virtually no influence on the freedom of choice index as long as the sample size remains above $8000 \approx e^9$. This is reassuring in two ways.

First, it means that the downsampling procedure does not introduce substantial error into the comparison between different years or subgroups for groups with 8000 or more households. When downsampling to lower values, the downsampling procedure creates an error (because the random draw of household matters) and a bias (because the diversity of consumption of individual households increasingly outweighs diversity of consumption across households).

Second, it means that if our household sample is a random, unbiased subsample of all households in the U.S., then the standard error of our index estimate is extremely low.[7] Thus, in the absence of any bias in the household sampling procedure, the standard error of the estimate is negligible. This is important since standard methods to estimate the standard error do not apply.

5.1 Growth in the Freedom of Choice index

In this subsection, we discuss the changes in the freedom of choice index from 2004 to 2017 as shown in Figure 4. Between 2004 and 2009 we observe almost no increase in the freedom of choice index, followed by a steep increase between 2009 and 2013 which slows during 2013 to 2017. We can interpret the change by calculating the corresponding change in the number of products purchased that would result in the same freedom of choice index value under the assumption that expenditure was evenly spent on all products. This can be calculated simply by exponentiating the freedom of choice index. In 2004, a measure of 11.25 corresponds to expenditure being equally distributed across approximately 77000 products. By 2009, this had increased to 79000 products. The steep increase in the measure up to 2013 corresponds to an increase to 97000 products. Up to 2017, there was a further increase to 103000 evenly distributed products. It is noteworthy that the logarithmic scale of the entropy index obscures the fact that a seemingly moderate increase of the index 2004 to 2017 corresponds to an increase in the number of different goods consumed by about one third.

There are many plausible reasons for the observed increase in the freedom of choice index. In the following subsections, we will show that the increase is a genuine welfare-relevant economic phenomenon and show that both changes in the supply of goods and the expenditure distribution of goods contributed to the increase in the freedom of choice index.

[7]This becomes clear by performing the following thought experiment. Suppose our sample of about 37 000 households in 2004 was the total number of households in the U.S.. Next, suppose our sample was a subsample of 20 000 households from which in turn we downsample to 10 000 households. Then it is almost irrelevant for the index estimate after downsampling *which* households were randomly selected in our sample of 20 000. Similarly, which 37 000 of the millions of U.S. households we draw has a negligible effect on our estimates as long as the sampling procedure is unbiased.

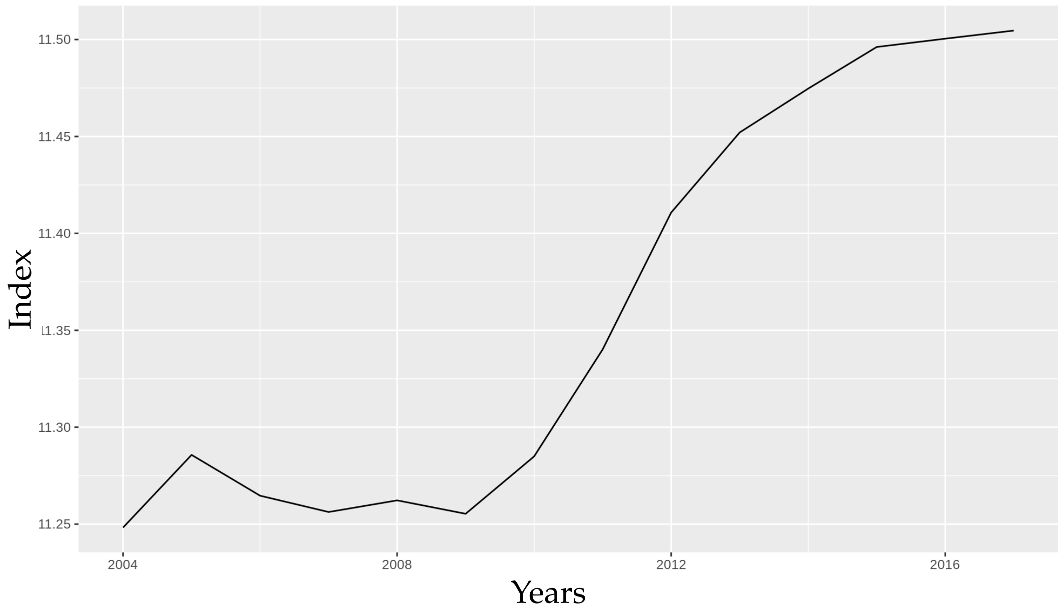

Figure 4: Freedom of Choice Index Over Time
Note: n=37783 per year.

Changes in the Number of Products Purchased

From the porperties of the entropy function follows that the increase in the freedom of choice index may be driven by firms offering more products and consumer expenditure distributing across a greater variety of goods or by a change in the demand of consumers who may distribute their expenditure more evenly across a fixed pool of goods.

To check whether the increase in the freedom of choice index is due to an increase in the number of products offered, we look at the total number of distinct products purchased by the households in our dataset. This corresponds to the case of $r \to 0$ of our characterized class of indices. While this number is of course only an estimate of the actual number of products available for purchase, it is a good indicator of the actual products offered since products that are not being purchased at all are likely driven out of the market.

As evident from Figure 5, the number of products purchased follows a pattern similar to that of the freedom of choice index; there is a steep increase in the total number of products purchased between 2009 and 2013. With an increase by approximately one fifth, the scale of the change is about the same as the change of the entropy-equivalent number of goods.

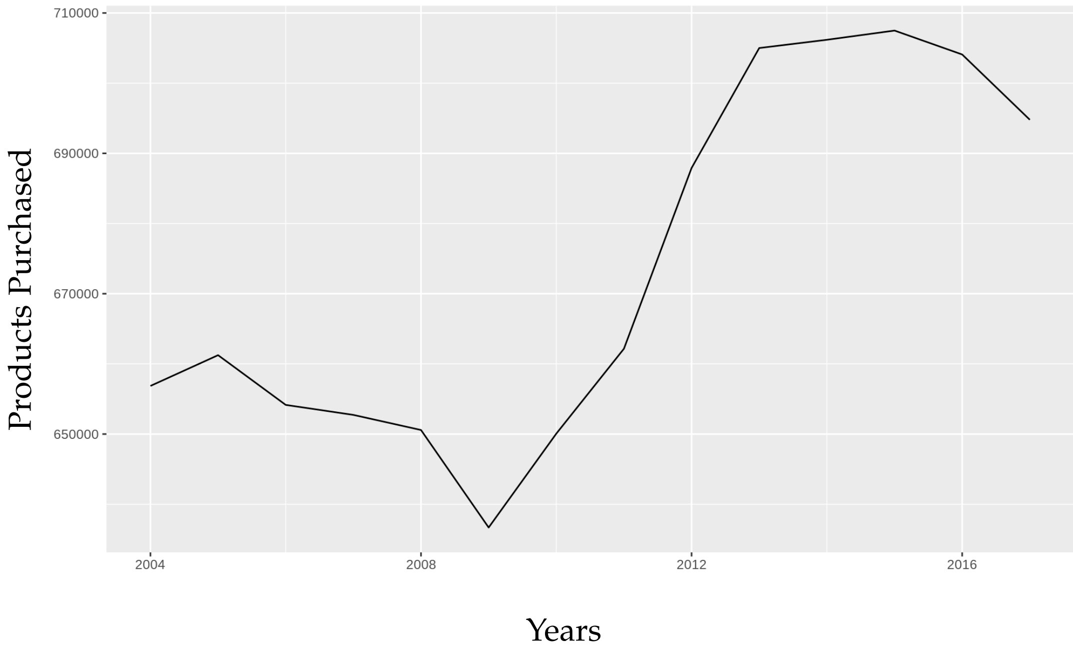

Figure 5: Number of Products Purchased by Year
Note: n=37783 per year.

However, for the duration from 2004 to 2009 and the duration from 2013 to 2017 instead of slight increases, we instead observe decreases in the total number of products purchased. This is a hint that the changes in the freedom of choice index cannot fully be explained by the number of products offered.

Changes in the Distribution of Expenditure

The freedom of choice index may also change if consumers distribute their expenditure more evenly across products. For example, it would be possible that with improved information technology, customers' ability to find products that fit their precise needs increases. Under a fixed distribution of needs and a fixed pool of products, this would naturally lead to an increase in the entropy of expenditure shares across products as niche products may receive a higher proportion of expenditure.

We therefore also examine the changes in the distribution of expenditure directly. To do this, in Figure 6 plot the rank-ordered cumulative distribution of expenditure across products, similar to a Lorenz-curve employed in the analysis of income inequality. Here we instead employ

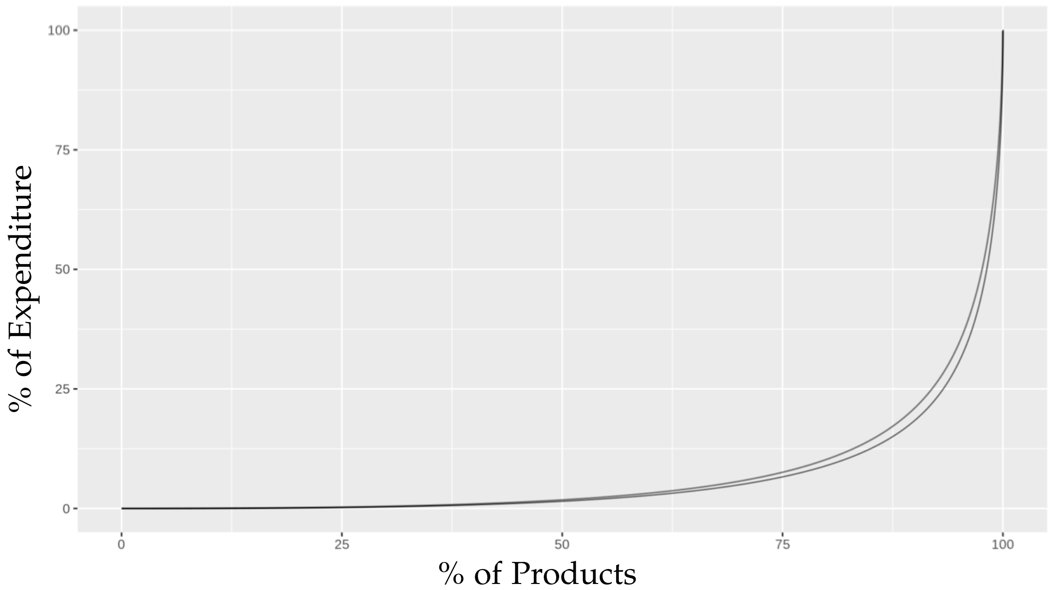

Figure 6: Product Expenditure Distribution Changes Between 2004 (lower line) and 2016 (upper line)
Note: n=37783 per year.

it to examine how unequal the expenditure is distributed across products. On the horizontal axis is the percentage of products ordered by total expenditure. The 25% mark accounts for the 25% of all products with the lowest expenditure. On the vertical axis is the cumulative percentage of expenditure spent on the products. The curves show how evenly the expenditure is distributed across products with a diagonal line representing a uniform distribution of expenditure across goods. As we can see, product expenditure is more evenly distributed in 2016 than in 2004. Thus, we conclude that not only the number of products offered by firms increased but also that the choices of consumers became more evenly distributed among those goods. Both effects increase the freedom of choice index. This leads to the natural question of how the freedom of choice index would have changed if the number of products had been identical in all years and only the expenditure distribution changed. This is plotted in Figure 7. The graph is obtained by estimating the freedom of choice index in every year on a random sample (without replacement) of products equal to the number of products offered in 2009. We see that as consumers allocated their expenditure more evenly across products, the freedom of choice index increased even if no additional products had been offered.

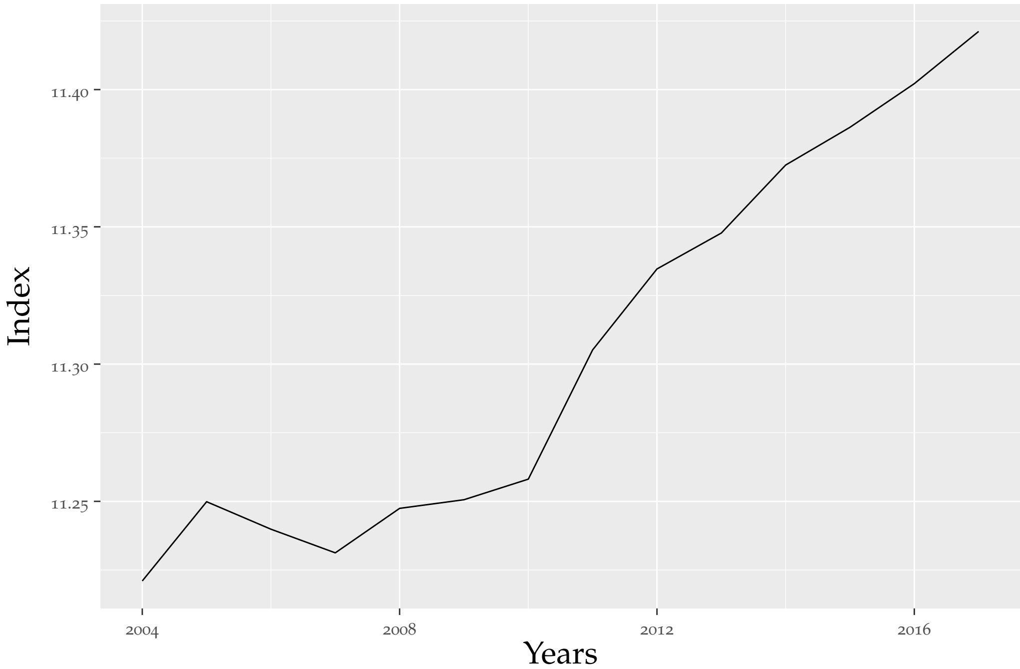

Figure 7: Hypothetical Freedom of Choice Index Holding Fixed the Number of Products

Note: n=37783, number of products is 638236.

Quantity Freedom of Choice Index

Another possible reason for an increase in the freedom of choice index is a change in price dispersion. If prices decrease of those goods that make up a larger proportion of expenditure, then the entropy of expenditure across products increases. We can verify that this is not the driving factor of the observed increase in the freedom of choice index by analyzing the entropy of the quantity of choices across goods, which we may call a quantity freedom of choice index. If the increase in the freedom of choice index was due to changes in prices only, then we should not observe an increase in the entropy of the distribution of the consumed quantities of goods.

We see from Figure 8 that indeed the quantity freedom of choice index changes reflect approximately the same pattern as the freedom of

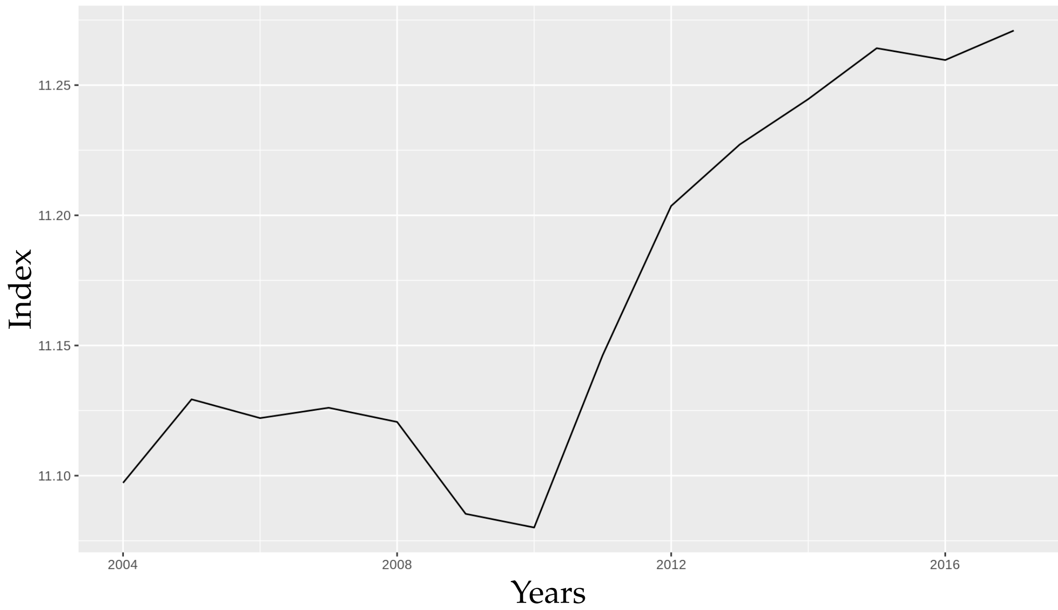

Figure 8: Quantity Freedom of Choice Index
Note: n=37783 per year.

choice index based on expenditure shares and therefore the increase in the freedom of choice index is not solely driven by changes in prices.

Growth of the Freedom of Choice Index for Subgroups

Since our sample slightly changes in household demographics over time, a possible reason for the observed increase in the freedom of choice index could be changes in demographics. This is especially true if demographic subgroups differ in their freedom of choice. One way of examining whether the increase in the freedom of choice index is due to changes in household demographics is to look at whether each demographic subgroup's freedom of choice index follows a similar pattern as that of the overall sample.

From Figure 9 we find within both age groups the same increase in the freedom of choice index as we have observed for the entire sample. Since the freedom of choice index applied to the age subgroups separately yields a slightly larger freedom of choice for young households, it is in principle possible that changes in the age composition of each group influences the freedom of choice. However, the slight changes in the age structure shown in Figure 1 exhibit a different pattern than the changes in the freedom of

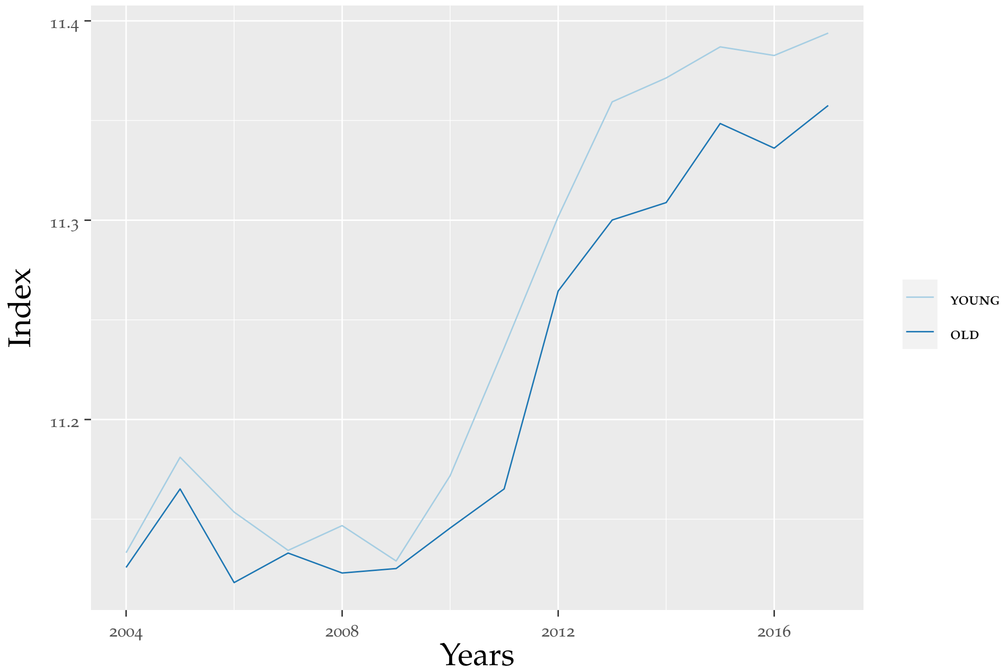

Figure 9: Freedom of Choice Index for Age Subgroups Over Time
Note: n=13979 for each subgroup.

choice index. Thus, the increase of the freedom of choice index after 2009 does not appear to be due to the change in age structure of the Nielsen Consumer Panel Data.

All subgroups with respect to race experienced an increase in the freedom of choice index over time as evident from Figure 10. Moreover, over time the percentage of WHITE households decreased and the percentage of ASIAN and AFRICAN AMERICAN households increased. As we can see from Figure 10, we should expect this demographic effect to decrease the freedom of choice index. Therefore, the increase of the freedom of choice index after 2009 is not due to changes in the racial composition of the Nielsen Consumer Panel Data.

Finally, with respect to income, we also find in Figure 11 that the freedom of choice index has experienced a similar growth pattern. We will discuss the difference between the two income groups in Section 5.3, when we discuss how income limits freedom of choice.

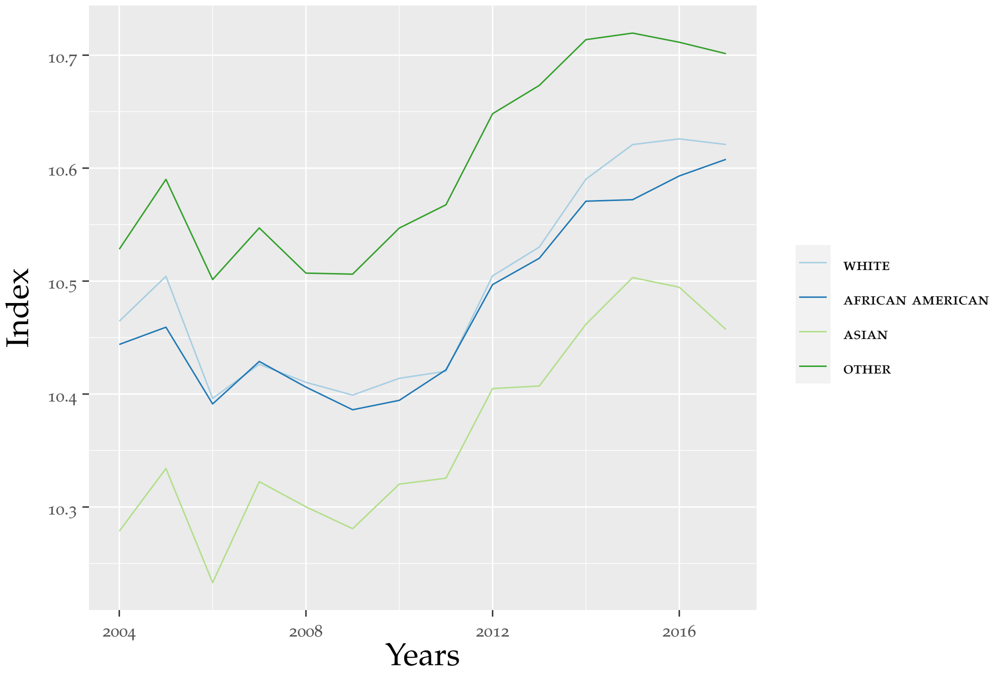

Figure 10: Freedom of Choice Index for Race Subgroups Over Time

Note: n=850 randomly drawn for each subgroup, values are averages of 100 calculations.

Estimation of the Freedom of Choice Index Using Product Descriptions

Suppose a small and a large retailer use their own, distinct UPCs for identical products. Then a gain in market share of the small retailer will lead to an increase in the freedom of choice index even if the relative expenditure shares across products are equal for both retailers. Similarly, reclassifying UPCs and changing the bundling of products may affect the index. We address this using the UPC description; instead of calculating the entropy across UPCs, we calculate the entropy across distinct words in the descriptions of the goods. The idea behind this is that even if two retailers assign different UPCs to the same product, they will generally still assign a very similar product description. Similarly, if goods are bundled, their description will generally contain words from the description of the two goods that are separately sold.

To calculate the freedom of choice index based on product descriptions, we first strip the UPC description of words regarding unit and amount and brand name. To do so, we employ the abbreviation identification

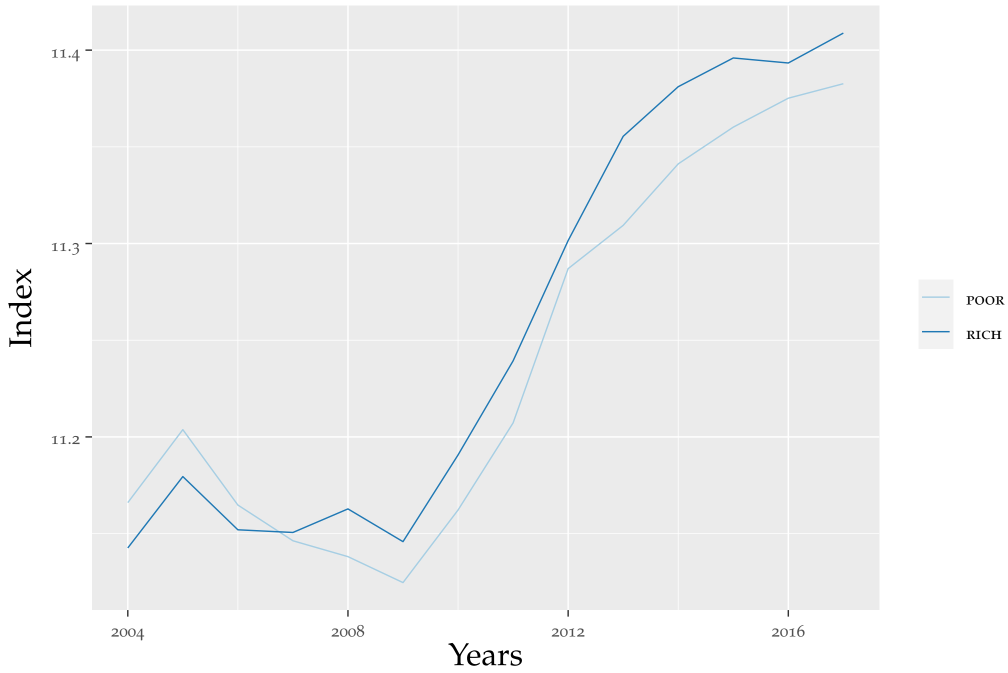

Figure 11: Freedom of Choice Index for Income Subgroups Over Time
Note: n=15000 for each subgroup.

algorithm of Schwartz and Hearst (2002). Next, for each module we separately estimate the entropy across words by assigning each word of a purchased UPS an equal share of the amount spent on the UPC. A (fictional) example would be a carton of free range eggs from the brand "Happy Eggs" with the description "HPPY E FR RNG EGG 12 PC". The algorithm would strip this to "FR RNG EGG" by removing the brand and size description. Next, we divide the expenditure on this product equally to the words "FR", "RNG", and "EGG" and calculate the freedom of choice index based on these expenditure values. It is important here that we must avoid conflating the word "FR" of free range eggs with the word "FR" of let's say a french wine. Therefore, the freedom of choice across dollar allocations to words is estimated separately for each module and averaged across modules weighted by expenditure. The results are shown in Figure 12. Naturally, there is a substantial decrease since product descriptions overlap across products. However, the pattern remains the same; we observe no change in the freedom of choice index up to 2009, after which the freedom of choice index increases up to 2017. We therefore conclude that the way UPCs are assigned to products is unlikely to be the main driver of the increase in the freedom of choice index.

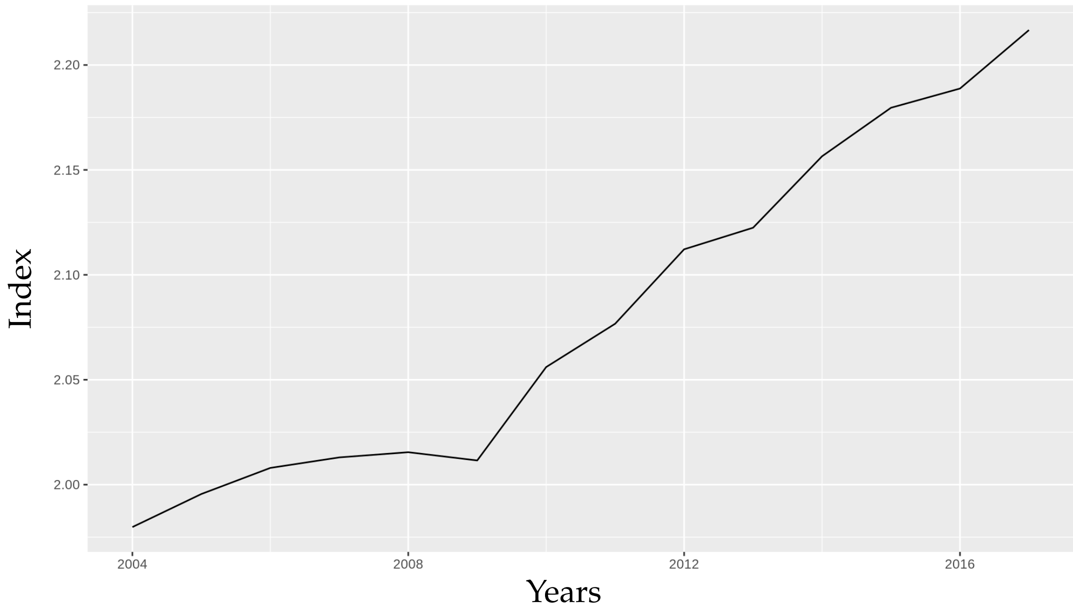

Figure 12: Freedom of Choice Index Applied To Expenditure Allocation to Product Description Words

Note: n=37783 per year.

Love of Variety

The increase in the freedom of choice index may be driven by households on average making more quantitatively diverse expenditure allocations or by households making more distinct expenditure allocations. The former cause would be classically associated with the love of variety of utility-maximizing households. Following the seminal work of Krugman (1979), the trade literature recognized that the availability of product varieties are welfare relevant and analyzed how limitations to trade may affect welfare through this channel. The literature commonly motivates this via a CES utility representation of a representative consumer in which the weights do not sum to one (e.g., Broda & Weinstein, 2006). However, the assumption of a single representative agent with a single set of fixed weights of course does not match the observed consumption patterns as we find great heterogeneity across households. It is therefore of interest to disentangle the love of variety of individual households from the freedom of choice we measure for the aggregate economy. For this, we calculate for every year for a random sample of 1000 households the Shannon (1948) entropy of the expenditure weights on household level.[8] Yearly averages and the 2.5 and 97.5 percentiles are plotted in Figure 13.

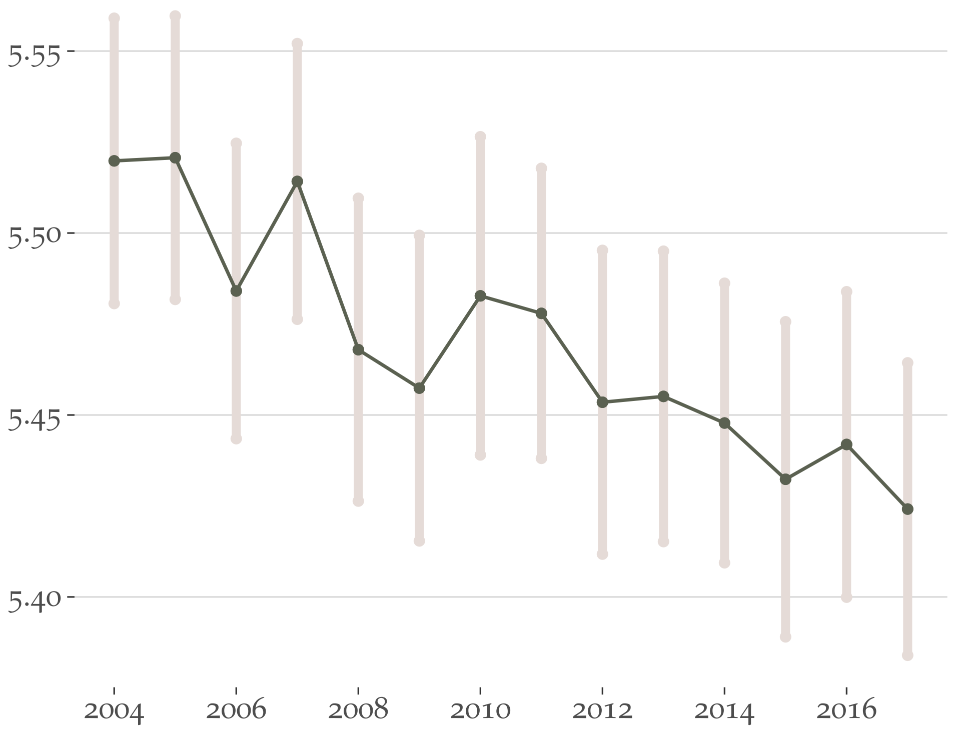

Figure 13: Average Love of Variety Measured by Expenditure Entropy
n = 1000 per year, bars represent a 95% confidence interval

As evident from Figure 13, contrary to the freedom of choice index, the love of variety of individual households indeed decreased over time. Thus, while on an aggregate scale, the diversity of products consumed increased over the duration of our data, expenditure became on average quantitatively less diverse for individual households. As both income and the availability of products increased, it is therefore plausible to assume that the love of variety at individual level is saturated; households'

[8]We could alternatively use H_r with r based on an empirical estimate of the elasticity of substitution across goods. We chose $r = 1$ for better comparability with the remaining results.

responded to increases in income by specializing the consumption to fewer goods. Our freedom of choice index captures instead the welfare effects of the diversity of consumption across households.

5.2 Determinants of the Freedom of Choice Index

Since we have axiomatically excluded inequality to play a role in our freedom of choice index, it is of interest to see whether there is inequality among different demographic groups in their freedom of choice. For this, we apply the index to the respective demographic groups and compare the results.

We use bootstrapping with 1000 i.i.d. bootstrap replications of 10% of the entire data. We sample without replacement as sampling with replacement would bias entropy estimates. In each bootstrap sample the number of households may differ between groups and we therefore need to downsample to the size of the smallest group. For the comparison of income and age groups we therefore simply downsample to the smaller group. For the subgroups with respect to race, as evident from Table 1, this process may lead to large variations in the number of households across the different bootstrap samples. We therefore instead draw 85 households from each race group out of the 10% bootstrap sample or draw another bootstrap sample if there are not 85 households of each race group. The resulting point estimates of the differences in the freedom of choice index make up our bootstrap dataset from which we calculate confidence intervals shown in Figure 14. In each graph the freedom of choice index value for the second listed subgroup is deducted from the first listed subgroup.

We find that african american households have a slightly higher freedom of choice index than white households. white and african american households have a substantially higher freedom of choice index than asian households. This may be driven by more heterogeneity in preferences of consumers in the former two groups. We also find that young households have a slightly higher freedom of choice index than old households. Finally, rich households have a slightly higher freedom of choice index than poor households.

It is also of interest whether more specific demographic groups have an especially large or small freedom of choice index. For example, a low income may only have a strong effect on freedom of choice for old households. Naturally, using the full interaction of age, income, and race

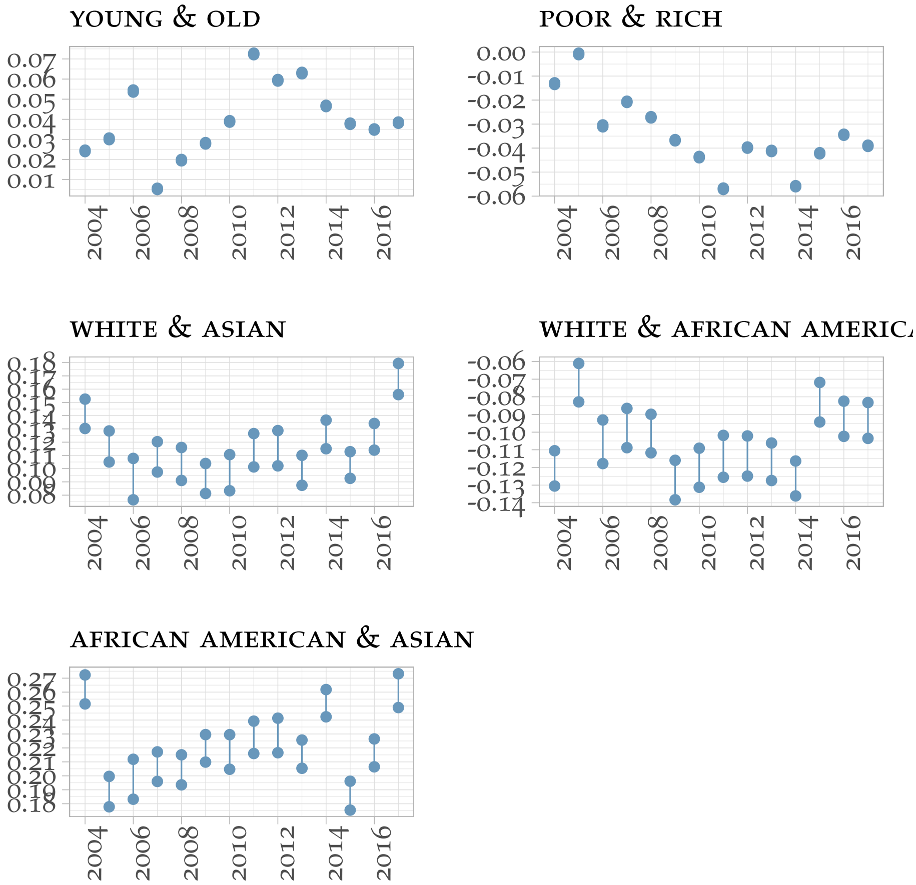

Figure 14: Confidence Intervals for the Freedom of Choice Index for Different Demographic Groups from 2004 to 2017

leaves us with relatively few households in each subgroup. We therefore use a group lasso regression with a similar bootstrap methodology as above. After drawing 10% of our data, we downsample to 10 households from each demographic group to calculate the freedom of choice index and apply the group lasso regression to the thereby constructed bootstrap sample. The possibility of obtaining standard errors comes at a cost however: if demographic groups differ in the love of variety of individual households, then a small number of households increasingly depends on the love of variety than the diversity of consumption across consumers. To select the demographic variables and interactions that are important, we use the group lasso estimation method (Yuan & Lin, 2006).

According to the group lasso procedure POOR, RICH, YOUNG AND POOR, YOUNG AND RICH, OLD AND POOR and OLD AND RICH do not provide effective explanations of the freedom of choice index. The effect of a higher income by itself does not seem to be associated with a higher freedom of choice index. We find that subgroups with respect to race and age exhibit a largely consistent picture with respect to differences of the freedom of choice index. With a few exceptions, OLD households generally have a lower freedom of choice than YOUNG households. The race subgroups of AFRICAN AMERICAN and OTHER households have a slightly higher freedom of choice index than WHITE and ASIAN households with the latter having the lowest index values. It is important to note that differences with respect to these subgroups may both stem from different degrees of heterogeneity of preferences within these groups or from an actual mismatch in the products offered with the preferences of the subgroups.

5.3 Income Inequality as a Limitation to Freedom of Choice

Income limitations to consumer freedom may arise from poorer households having less freedom of choice or from income determining the choices of households. Both represent limitations to freedom of choice – the former by some agents having more freedom of choice than others and the latter by agents not being in full control of what they choose. With respect to the former, we have already seen that the diversity of expenditure is only slightly lower for POOR households. An analysis of the latter checks for a limitation of the self-determination of consumption of households by income.

Figure 16 measures to what extent income determines purchases. This is done using mutual information which is cardinally comparable to the freedom of choice index if $r = 1$. Under the assumption that agents are not morally responsible for their income, the mutual information between income and consumption choices creates an upper bound for the freedom of choice index in Rommeswinkel (2019a), an extension of the entropy measure. The higher the mutual information between income and purchases, the lower the bound.

In order to eliminate effects from our household size adjustment of income, we focus on households with two household members. We find that the mutual information between income group categories and purchases is on average about 0.09 for two person households. With more precise income data, this number may of course increase. The maximum

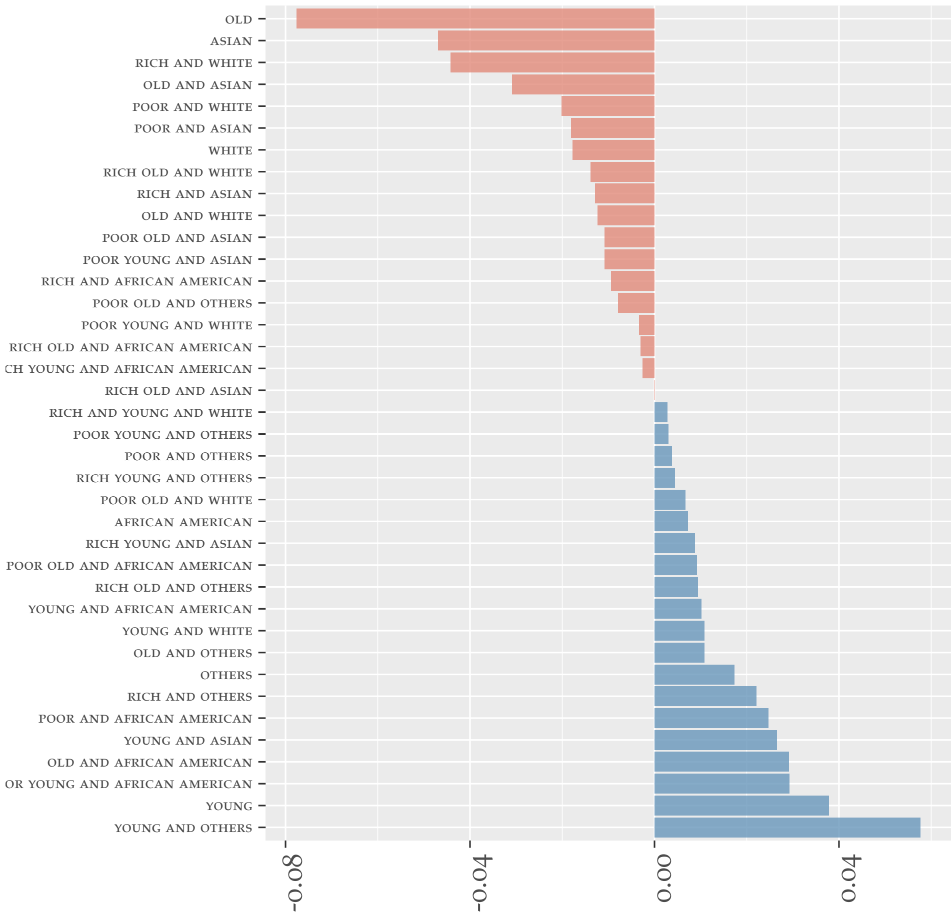

Figure 15: Difference of the Freedom Of Choice Index of Various Demographic Groups from the Average

possible value given our income categories can be calculated to be about 3.5 in which case each income category fully determines a set of products that is only purchased by households of this income class. Since the freedom of choice index and mutual information are additively comparable, we can interpret the limitation as roughly one third of the increase between 2004 and 2017.

Figure 11 plots our freedom of choice index for approximately the top half and lowest half of income. As expected, the freedom of choice index differs across the two groups. We find that the freedom of choice index was almost identical for the two groups in the first years of the sample. Toward the end of the sampling period we find a difference in the freedom

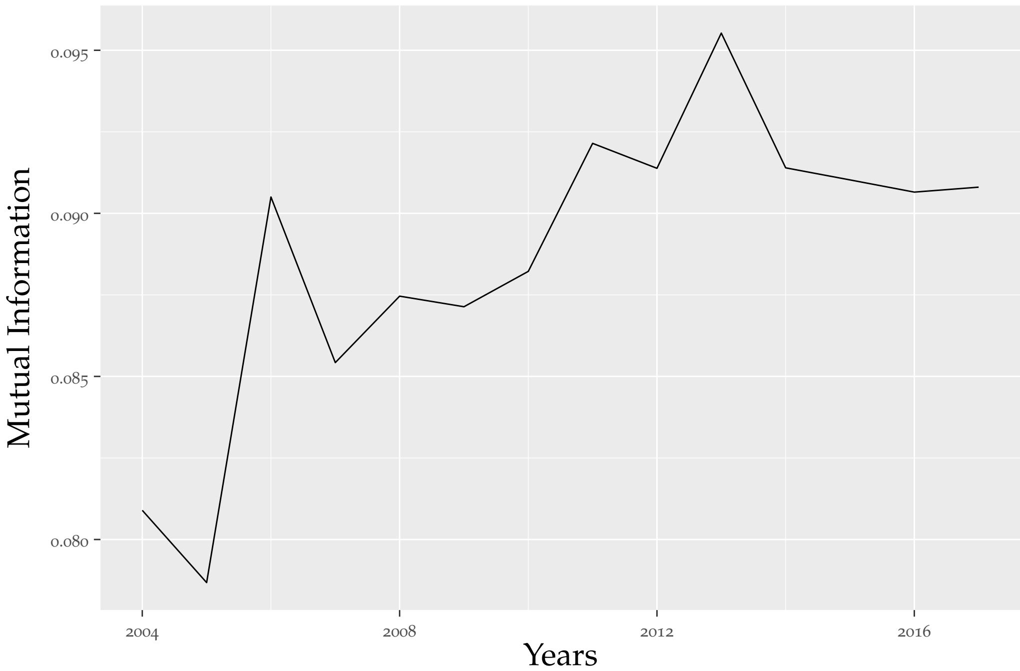

Figure 16: Mutual Information Between Income Groups and Expenditure for Households of Size 2

Note: n=37783.

of choice index between the top and lowest income group of about .06. This suggests that consumer markets provide a substantial diversity of choice for all income groups: the main limitation of freedom of choice arises from income determining what consumers purchase and not from a lack of diversity of choice.

6 Discussion

Classically, welfare indices face a tradeoff between parsimony and potential misspecification. For example, a Sato-Vartia quantity index can easily be estimated and has few parameters. It is commonly motivated as an index of utility of a representative consumer with a CES utility representation. However, from consumption data, it is easy to reject both the assumption of a representative agent and of identical substitutability of all goods, implying a misspecified model of welfare. Similarly, the love of variety literature assumes a representative household with CES utility to motivate welfare-relevant effects of the number of available products. Our data shows that over the duration of our sample the households decreased the diversity of products purchased while the diversity of purchases across households increased. Thus, any utilitarian index that captures the welfare gains of the increase of freedom of choice would require heterogeneous preferences. It is an open question whether a sufficiently parsimonious utilitarian index exists that is based on preferences that match choice behavior in the data. In this paper, we therefore returned to the method of axiomatically deriving an index without imposing utility-maximizing households. The freedom of choice literature provides a theoretical underpinning for an intrinsic value of the index we characterized.

Empirically, we observe that the freedom of choice index has been stagnant between 2004 and 2009 but has experienced a substantial increase between 2010 and 2017. We showed that the change in the index reflects actual changes in the economy and is driven by an increase in the available goods and a quantitatively more diverse purchasing behavior of consumers.

There are many avenues for further research. An important question is what the macroeconomic and behavioral determinants of the freedom of choice index are, particularly the causes of the increase in the freedom of choice index we observed. It is also noteworthy that the production of a greater variety of goods uses resources due to reduced economies of scale and a greater complexity. Thus, a time period of reduced economic growth of income accompanied by a growth in the freedom of choice may easily be misinterpreted as a reduced economic growth. Economic growth in a wide sense may therefore also occur by diversification of production which is invisible to standard income measures. It is thus crucial to better understand the interplay of policies, the behavior of economic agents, and freedom of choice.

Acknowledgements

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The conclusions drawn from the Nielsen data are those of the researcher(s) and do not reflect the views of Nielsen. Nielsen is not responsible for, had no role in, and was not involved in analyzing and preparing the results reported herein.

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A Proofs

A.1 Lemma 1

Proof. By the two invariance axioms, $(q, e) \sim (\alpha q, \alpha e) \sim (\alpha q, e)$ for a positive real number α. By Weak Subgroup Decomposability,

$$(0_g q, 0_g e) \sim \left(\frac{1}{\sum_h q_g^h} 0_g q, 0_g e \right) \Leftrightarrow (q_g q', e) \sim \left(\left(\frac{1}{\sum_h q_g^h} q \right)_g q', e \right) \tag{A.6}$$

Applying this step iteratively to all g, we obtain that

$$(q, e) \sim \left(\left(\frac{1}{\sum_{c'} q_{c'g}} q_g \right)_{g \in \mathcal{G}}, e \right). \tag{A.7}$$

By Inflation Invariance, we have that

$$(q, e) \sim \left(\left(\frac{1}{\sum_{c'} q_{c'g}} q_g \right)_{g \in \mathcal{G}}, \frac{1}{\sum_{c,g} e_{cg}} e \right). \tag{A.8}$$

□

A.2 Lemma 2

Proof. By Inequality Invariance, we may redistribute the entire consumption and expenditure of every good to the first consumer such that all other consumers have zero quantity and zero expenditure and the first consumers' quantity and expenditure are the respective sums across all consumers. By the previous lemma, we may divide the expenditure data by total expenditure without changing the ranking in the index. Moreover, we may divide the consumers' quantities of a good by the total amount consumed of that good without changing the index. It follows that quantities are irrelevant as the first consumer consumes a share of 1 of each good. We therefore have that $(q, e) \sim (\bar{q}, \bar{e})$. where $\bar{e}_{1g} = \frac{\sum_{c \in \mathcal{C}} e_{gc}}{\sum_{c \in \mathcal{C}, g \in \mathcal{G}} e_{gc}}$ and $\bar{e}_{cg} = 0$ for all consumers $c \neq 1$. We then define $s_{q,e} \succsim^* s_{q',e'}$ if and only if $(\bar{q}, \bar{e}) \succsim (\bar{q}', \bar{e}')$. □

A.3 Theorem 1

We have established that the relation can be reduced to a relation over the expenditure shares. We next prove that Weak Subgroup Decomposability of the index implies Disjoint Independence on the expenditure share relation. From there the result follows from Theorem 2.

Suppose $s_{q,e}$ and $s_{q',e'}$ both have a support that is disjoint from that of $s_{q'',e''}$.

$$s_{q,e} \succsim^* s_{q',e'} \tag{A.9}$$

$$\Leftrightarrow \quad (q, \frac{1}{\sum_{c,g} e_{cg}} e) \succsim (q', \frac{1}{\sum_{c,g} e_{cg}} e') \tag{A.10}$$

$$\Leftrightarrow \quad (q, \frac{\alpha}{\sum_{c,g} e_{cg}} e) \succsim (q', \frac{\alpha}{\sum_{c,g} e_{cg}} e') \tag{A.11}$$

$$\Leftrightarrow \quad ((q'')_{supp(s_{q'',e''})} q, (\frac{1-\alpha}{\sum_{c,g} e''_{cg}} e'')_{supp(s_{q'',e''})} (\frac{\alpha}{\sum_{c,g} e_{cg}} e)) \tag{A.12}$$

$$\succsim ((q'')_{supp(s_{q'',e''})} q, (\frac{1-\alpha}{\sum_{c,g} e''_{cg}} e'')_{supp(s_{q'',e''})} (\frac{\alpha}{\sum_{c,g} e_{cg}} e)) \tag{A.13}$$

$$\Leftrightarrow \quad \alpha s_{q,e} \oplus (1-\alpha) s_{q'',e''} \succsim^* \alpha s_{q',e'} \oplus (1-\alpha) s_{q'',e''} \tag{A.14}$$

The first step follows from Lemma 2, while the second follows from Inflation Invariance. The third step follows from applying Weak Subgroup Decomposability twice and the last step again from 2.

A.4 Theorem 2

Instead of working with mass functions, it will be convenient to extend these to measures. Since our mass functions $p \in \Delta\mathcal{G}$ are defined on a finite set, it is straightforward to extend these into measures on the power set by denoting for a set $\mathcal{G}' \subseteq \mathcal{G}$, $p(\mathcal{G}') = \sum_{g \in \mathcal{G}'} p(g)$.

We consider a partition of the set of goods $\mathcal{G}$ into three sets $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$, each containing an essential pair of goods. Note, that any element e of $\Delta\mathcal{G}$ can be expressed in a compounded form such as $e = \alpha a \oplus (1-\alpha)(\frac{\beta}{1-\alpha} b \oplus \frac{\gamma}{1-\alpha} c)$ where $supp(a) \subseteq \mathcal{A}$, $supp(b) \subseteq \mathcal{B}$, and $supp(c) \subseteq \mathcal{C}$ and $\alpha = e(\mathcal{A})$, $\beta = e(\mathcal{B})$, and $\gamma = e(\mathcal{C})$.

For fixed α, β, γ, $\succsim$ yields a complete, transitive, and continuous relation on the product space $\Delta\mathcal{A} \times \Delta\mathcal{B} \times \Delta\mathcal{C}$ that fulfills joint independence of the factors. By Gorman (1968), we obtain a representation of the form:

$$U(e) = h_{\mathcal{A},\mathcal{B},\mathcal{C}}(f_{\mathcal{A},\mathcal{B},\mathcal{C}}(a,\alpha,\beta) + g_{\mathcal{A},\mathcal{B},\mathcal{C}}(b,\alpha,\beta) + h_{\mathcal{A},\mathcal{B},\mathcal{C}}(c,\alpha,\beta),\alpha,\beta) \quad \text{(A.15)}$$

where $\gamma = 1 - \alpha - \beta$ can be omitted. By obtaining an analogous representation for a different partitioning of $\mathcal{G}$ into $\mathcal{A}$, $\mathcal{B}'$, and $\mathcal{C}'$, we obtain:

$$\begin{aligned} U(e) =& h_{\mathcal{A},\mathcal{B},\mathcal{C}}(f_{\mathcal{A},\mathcal{B},\mathcal{C}}(a,\alpha,\beta) + g_{\mathcal{A},\mathcal{B},\mathcal{C}}(b,\alpha,\beta) + h_{\mathcal{A},\mathcal{B},\mathcal{C}}(c,\alpha,\beta),\alpha,\beta) \\ =& T(h_{\mathcal{A},\mathcal{B}',\mathcal{C}'}(f_{\mathcal{A},\mathcal{B}',\mathcal{C}'}(a,\alpha,\beta') + g_{\mathcal{A},\mathcal{B}',\mathcal{C}'}(b,\alpha,\beta') + h_{\mathcal{A},\mathcal{B}',\mathcal{C}'}(c,\alpha,\beta'),\alpha,\beta')) \end{aligned} \quad \text{(A.16)}$$

where it is without loss of generality to assume that T is the identity transformation $T : x \mapsto x$. For fixed α, β, β', we have two additively separable representations on $\Delta\mathcal{A}]$ and $\Delta\mathcal{B} \cup \mathcal{C}]$. By the uniqueness[9] of additive representations, it follows that $h^{-1}_{\mathcal{A},\mathcal{B},\mathcal{C}}(h_{\mathcal{A},\mathcal{B}',\mathcal{C}'}(x,\alpha,\beta'),\alpha,\beta) = A(\alpha,\beta,\alpha',\beta')x + B(\alpha,\beta,\alpha',\beta')$ is affine and $f_{\mathcal{A},\mathcal{B}',\mathcal{C}'}(a,\alpha,\beta')$ is an affine transformation of $f_{\mathcal{A},\mathcal{B},\mathcal{C}}(a,\alpha,\beta)$. It follows that $f_{\mathcal{A},\mathcal{B},\mathcal{C}}(a,\alpha,\beta) = A_{\mathcal{A},\mathcal{B},\mathcal{C}}(\alpha,\beta)\bar{f}_{\mathcal{A}}(a,\alpha) + B_{\mathcal{A},\mathcal{B},\mathcal{C}}(\alpha,\beta)$ where we make use of the fact that the choice of the partitioning into $\mathcal{B}'$ and $\mathcal{C}'$ is irrelevant. Analogous arguments can be made for the remaining additive components. In summary,

$$U(e) = h_{\mathcal{A},\mathcal{B},\mathcal{C}}(A_{\mathcal{A},\mathcal{B},\mathcal{C}}(\alpha,\beta)\bar{f}_{\mathcal{A}}(a,\alpha) + A_{\mathcal{A},\mathcal{B},\mathcal{C}}(\alpha,\beta)\bar{f}_{\mathcal{B}}(b,\beta) \quad \text{(A.17)}$$

$$+ A_{\mathcal{A},\mathcal{B},\mathcal{C}}(\alpha,\beta)\bar{f}_{\mathcal{C}}(c,1-\alpha-\beta),\alpha,\beta) \quad \text{(A.18)}$$

where without loss of generality we assumed that the additive transformation is zero for each of the components. We can redefine $h_{\mathcal{A},\mathcal{B},\mathcal{C}}(A_{\mathcal{A},\mathcal{B},\mathcal{C}}(\alpha,\beta)x,\alpha,\beta) = \bar{h}_{\mathcal{A},\mathcal{B},\mathcal{C}}(x,\alpha,\beta)$. Further, note that f must be increasing in the utility of a,

[9] If U and $\bar{U}$ are additive representations on a connected, separable product space $X \times Y$, then they are affine transformations of another and the component functions are unique up to joint linear transformations and component-specific additive transformations.

thus we can redefine $\bar{f}$ such that,

$$U(e) = h_{\mathcal{A},\mathcal{B},\mathcal{C}}(\bar{f}_{\mathcal{A}}(U(a),\alpha) + \bar{f}_{\mathcal{B}}(U(b),\beta) + \bar{f}_{\mathcal{C}}(U(c), 1-\alpha-\beta), \alpha, \beta) \tag{A.19}$$

For all e such that $\gamma = 0$, the above representation must be invariant in $U(c)$ and we therefore obtain the following representation by taking limits:

$$\begin{aligned} U(e) =& h_{\mathcal{A},\mathcal{B},\mathcal{C}}(\bar{f}_{\mathcal{A}}(U(a),\alpha) + \bar{f}_{\mathcal{B}}(U(b),\beta) + \bar{f}_{\mathcal{C}}(U(c),0), \alpha, \beta) && \text{(A.20)} \\ \equiv& h'_{\mathcal{A},\mathcal{B},\mathcal{C}}(\bar{f}_{\mathcal{A}}(U(a),\alpha) + \bar{f}_{\mathcal{B}}(U(b), 1-\alpha), \alpha) && \text{(A.21)} \end{aligned}$$

Partitioning $\mathcal{B}$ into $\mathcal{B}'$ and $\mathcal{B}''$, we obtain a representation for $U(b)$, which we can plug back into (A.22):

$$\begin{aligned} U(e) =& h_{\mathcal{A},\mathcal{B},\mathcal{C}}(f_{\mathcal{A}}(U(a),\alpha) \\ &\quad + f_{\mathcal{B}}(h'_{\mathcal{B}',\mathcal{B}'',\mathcal{A}\cup\mathcal{C}}(f_{\mathcal{B}'}(U(b'),\beta') + f_{\mathcal{B}''}(U(b''), 1-\beta'), \beta'), 1-\alpha), \alpha) \\ =& h_{\mathcal{A}\cup\mathcal{C},\mathcal{B}',\mathcal{B}''}(\bar{f}_{\mathcal{A}\cup\mathcal{C}}(U(a),\alpha) + \bar{f}_{\mathcal{B}'}(U(b'),\beta\beta') \\ &\quad + \bar{f}_{\mathcal{B}''}(U(b''), 1-\alpha-\beta\beta'), \alpha, \beta\beta') && \text{(A.22)} \end{aligned}$$

Since we know that for fixed α, β, β', by the uniqueness of additive representations over a and b, $h^{-1}_{\mathcal{A},\mathcal{B},\mathcal{C}}(h_{\mathcal{A}\cup\mathcal{C},\mathcal{B}',\mathcal{B}''}(x,\alpha,\beta\beta'),\alpha,\beta)$ is affine in x, we have that $f_{\mathcal{B}}(h'_{\mathcal{B}',\mathcal{B}'',\mathcal{A}\cup\mathcal{C}}(x,\beta'), 1-\alpha) = F(\alpha)x + H(\alpha)$ is also affine in x.

Note that for some arbitrary fixed β', we can assume that $h'_{\mathcal{B}',\mathcal{B}'',\mathcal{A}\cup\mathcal{C}}$ is affine. If for some U this is not the case, then it is the case after applying the continuous monotone transformation $(h'_{\mathcal{B}',\mathcal{B}'',\mathcal{A}\cup\mathcal{C}})^{-1}(\cdot,\beta')$ for the fixed β'. Yet, it then follows that $h_{\mathcal{A},\mathcal{B},\mathcal{C}}$ and $f_{\mathcal{B}}$ are also affine in their first arguments, independent of the choice of α and β. From this we can derive that all other functions $h_{\dots}$ must be also be affine in their first argument. We obtain the representation:

$$\begin{aligned} U(e) =& (f_{\mathcal{A}}(U(a),\alpha) + U(b)F(1-\alpha) + H(1-\alpha))\, A(\alpha) + B(\alpha) \\ =& h_{\mathcal{A}',\mathcal{A}'',\mathcal{B}\cup\mathcal{C}}(\bar{f}_{\mathcal{A}'}(U(a'),\alpha\alpha') + \bar{f}_{\mathcal{A}''}(U(a''),\alpha(1-\alpha')) \\ &\quad + \bar{f}_{\mathcal{B}\cup\mathcal{C}}(U(b),\alpha\alpha',\alpha(1-\alpha')),\alpha',\alpha'') && \text{(A.23)} \end{aligned}$$

for e in which $\gamma = 0$. We now argue that $f_{\mathcal{A}}$ is also affine in its first argument. First, note that $h_{\mathcal{A},\mathcal{A}',\mathcal{B}\cup\mathcal{C}}$ must be affine. Second, $U(a)$ is a limiting case of the second line of (A.23) and therefore additively separable in a' and a'' if $h_{\mathcal{A},\mathcal{A}',\mathcal{B}\cup\mathcal{C}}$ is affine. It then follows that $f_{\mathcal{A}}$ is affine in its first argument. We have therefore that if for some $f_{\mathcal{B}}$ we obtain affinity, then we can extend this to an arbitrary $f_{\mathcal{A}}$ with $\mathcal{A} \cap \mathcal{B} = \emptyset$. Choosing any partition $\mathcal{P}$ of $\mathcal{G}$ we thus obtain the representation:

$$U(e) = \sum_{\mathcal{A} \in \mathcal{P}} U(a) F_{\mathcal{A}}(e(\mathcal{A})) + H_{\mathcal{A}}(e(\mathcal{A})) \tag{A.24}$$

From the partition $\{\mathcal{A}, \mathcal{G} - \mathcal{X}\}$ and its refinement $\{\mathcal{A}', \mathcal{A}'', \mathcal{G} - \mathcal{A}\}$, we then obtain

$$U(a') F_{\mathcal{A}}(\alpha) F_{\mathcal{A}'}(\alpha') + \ldots = U(a') F_{\mathcal{A}}(\alpha\alpha') + \ldots \tag{A.25}$$

where $\ldots$ denotes components that do not depend on $U(a')$. The above equation is only maintained for small changes of $U(a')$ if

$$F_{\mathcal{A}}(\alpha) F_{\mathcal{A}'}(\alpha') = F_{\mathcal{A}}(\alpha\alpha') \tag{A.26}$$

Taking logs on both sides gives us:

$$\ln F_{\mathcal{A}}(\alpha) + \ln F_{\mathcal{A}'}(\alpha') = \ln F_{\mathcal{A}'}(\alpha\alpha') \tag{A.27}$$

which is a Pexider-like equation with the solution:

$$F_{\mathcal{A}}(\alpha) = \alpha^{\psi} \tag{A.28}$$

$$F_{\mathcal{A}'}(\alpha') = (\alpha')^{\psi} \tag{A.29}$$

Thus, the representation simplifies to:

$$U(e) = \sum_{\mathcal{A} \in \mathcal{P}} U(a) e(\mathcal{A})^{\psi} + H_{\mathcal{A}}(e(\mathcal{A})) \tag{A.30}$$

From different partitions and their representations we then obtain:

$$U(a)\alpha^{\psi} + H_{\mathcal{A}}(\alpha) + H_{\mathcal{B}\cup\mathcal{C}}(1-\alpha) \tag{A.31}$$

$$+ (1-\alpha)^{\psi}\left(\frac{\beta}{1-\alpha}U(b) + H_{\mathcal{B}}\left(\frac{\beta}{1-\alpha}\right) + \frac{\gamma}{1-\alpha}U(c) + H_{\mathcal{C}}\left(\frac{\gamma}{1-\alpha}\right)\right) \tag{A.32}$$

$$= U(b)\alpha^{\psi} + H_{\mathcal{B}}(\beta) + H_{\mathcal{A}\cup\mathcal{C}}(1-\alpha) \tag{A.33}$$

$$+ (1-\alpha)^{\psi}\left(\frac{\alpha}{1-\beta}U(a) + H_{\mathcal{A}}\left(\frac{\alpha}{1-\beta}\right) + \frac{\gamma}{1-\beta}U(c) + H_{\mathcal{C}}\left(\frac{\gamma}{1-\beta}\right)\right) \tag{A.34}$$

Defining $\bar{H}_{\mathcal{D}}(\delta) = H_{\mathcal{D}}(\delta) + H_{\mathcal{G}-\mathcal{D}}(\delta)$ for all subsets $\mathcal{D} \subseteq \mathcal{G}$, we obtain after cancelling terms:

$$\bar{H}_{\mathcal{A}}(\alpha) + (1-\alpha)^{\psi}\left(\bar{H}_{\mathcal{B}}\left(\frac{\beta}{1-\alpha}\right)\right) \tag{A.35}$$

$$= \bar{H}_{\mathcal{B}}(\beta) + (1-\alpha)^{\psi}\left(\bar{H}_{\mathcal{A}}\left(\frac{\alpha}{1-\beta}\right)\right) \tag{A.36}$$

This is the generalized fundamental equation of information with the continuous solutions (Ebanks et al., 1987):

$$\bar{H}_{\mathcal{D}}(\delta) = \begin{cases} \phi\left(\delta \ln \delta + (1-\delta)\ln(1-\delta)\right) + \delta\theta_{\mathcal{D}} + \zeta_{\mathcal{D}} & \psi = 1 \\ \phi\left(\zeta_{\mathcal{D}}\delta^{\psi} + (1-\delta)^{\psi} - 1\right) & \psi \neq 1 \end{cases} \tag{A.37}$$

Plugging this solution into (A.30) and gathering terms gives us the desired representation up to monotone transformations.

A.5 Axiomatic Foundation for the Case r=1

Axiom 1 (Cardinal Consistency)**.** If a, b, c are disjoint, $\bar{a}, \bar{b}, \bar{c}$ are disjoint, $a \sim b$ and $\bar{a} \sim \bar{b}$, then

$$(1-\mu)(\lambda a \oplus (1-\lambda)b) \oplus \mu c \succsim (1-\mu)(\lambda \bar{a} \oplus (1-\lambda)\bar{b}) \oplus \mu \bar{c} \tag{A.38}$$

$$\Leftrightarrow \quad (1-\mu)a \oplus \mu c \succsim (1-\mu)\bar{a} \oplus \mu \bar{c} \tag{A.39}$$

The axiom guarantees that the value on every subset of goods is cardinally comparable to the value on a different subset of goods. On the LHS of the preference, in the first line the value of a is achieved both on the set of goods in $supp(a)$ and $supp(b)$. In the second line, the value is only generated on $supp(a)$ but with the same overall expenditure proportion. On the RHS, the value is generated initially on $supp(\bar{a})$ and $supp(\bar{b})$ (which need not be the same as $supp(a)$ and $supp(b)$) and in the second line only on $supp(a)$, but with the same overall expenditure weight. Cardinal Consistency states that switching from the first to the second line the partial order may not be reversed. This essentially means that the cardinal value generated on a proportion of expenditure is unaffected by the set of goods on which it occurs.

We obtain the following Corollary:

Corollary 1. *$r = 1$ in Theorem 2 holds if and only if $\succsim$ fulfills Cardinal Consistency.*

The proof of this result is trivial. It is straightforward to show that the partial order induced by the Shannon entropy fulfills Cardinal Consistency. For the reverse implication, it is easy to show that the case of $r \neq 1$ violates Cardinal Consistency.

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